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2005 J. Phys. A: Math. Gen. 38 6603

(http://iopscience.iop.org/0305-4470/38/29/014)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 6603–6611 doi:10.1088/0305-4470/38/29/014

On decoherence in noncommutative plane with perpendicular magnetic field

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Received 24 March 2005
Published 6 July 2005
Online at stacks.iop.org/JPhysA/38/6603

Abstract

In the last few years noncommutative quantum mechanics has been investigated intensively. We consider the influence of magnetic field on decoherence of a system in the noncommutative quantum plane. Particularly, we point out a model in which the magnetic field allows *in situ* dynamical control of decoherence as well as, in principle, observation of noncommutativity.

PACS numbers: 11.10.Nx, 03.65.Yz

1. Introduction

Thinking about a possible solution of the problem of ultraviolet divergences, already in the 1930s Heisenberg conjectured that position coordinates might be noncommutative (NC). Snyder [1] was the first who started to develop this idea systematically in 1947. An intensive interest in NC quantum theories emerged after observation of noncommutativity in string theory with D-branes in 1998. Most of the research has been done in the framework of NC field theory (for reviews, see, e.g. [2, 3]). NC quantum mechanics (NCQM) has been also actively investigated. It enables construction of simple NC models which have relevance to concrete phenomenological systems and can be regarded as the corresponding one-particle nonrelativistic sector of NC quantum field theory. Although spacetime uncertainties have their origin in string theory and quantum gravity at the Planck scale, they may play a significant role in some important quantum-mechanical phenomena. An experimental observation of noncommutativity at the non-relativistic level, would be a striking physical event. To this end, many quantum-mechanical effects (Aharonov–Bohm effect, lowest Landau level, fractional quantum Hall effect, . . .) in a NC background have been studied and the corresponding influences were calculated (see, e.g., [2–4] and references therein). Quantum decoherence is one of the new phenomena, of fundamental and practical importance, which combined with noncommutativity becomes more intriguing and might cause much greater interest.

Here, we investigate the possible influence of noncommutativity on the occurrence of the so-called decoherence effect [5–10]. Quantum decoherence is sometimes considered to be a fundamental physical basis for the 'transition from quantum to classical' [5, 10], i.e. the (semi)classical behaviour of the (open) quantum systems (that cannot in principle be described by the Schrödinger equation). Actually, the environment induces the effective *superselection rules* for an open system by destroying the linear ('coherent') superpositions of certain states (the so-called 'pointer basis' states) of the open system [6]. The non-interfering states thus appear 'objectively' to be present for an observer, very much like one would expect for the macroscopic (classical) systems [5, 10].

Quantum decoherence is a distinguished theory of modern quantum mechanics. It is therefore *per se* interesting to investigate the occurrence of decoherence in NCQM. At first sight, it is unlike to be able to observe a classical-like effect which has its origin in the Planck scale quantum physics. However, this should not be the case for NCQM with decoherence. As a result, we find some interesting consequences of noncommutativity for a simple model of the decoherence theory. Particularly, we point out a situation in which the experimenter may *in situ* (at will) control the decoherence, i.e. by an external magnetic field induce wanted behaviour of the open system, in principle allowing the observation of the phase space noncommutativity. To our best knowledge, the influence of the noncommutativity on decoherence has not been explored so far.

2. A phase space noncommutativity

We use here NCQM which is based on the following algebra for the position and the momentum coordinates [4]:

$$[\hat{x}_a, \hat{p}_b] = i\hbar(\delta_{ab} - \theta_{ac}\sigma_{cb}/4), \qquad [\hat{x}_a, \hat{x}_b] = i\hbar\theta_{ab}, \qquad [\hat{p}_a, \hat{p}_b] = i\hbar\sigma_{ab}, \tag{1}$$

where $(\theta_{ab}) = \Theta$ and $(\sigma_{ab}) = \Sigma$ are the antisymmetric matrices with constant elements. It allows simple reduction to the usual algebra

$$[\hat{q}_a, \hat{k}_b] = i\hbar\delta_{ab}, \qquad [\hat{q}_a, \hat{q}_b] = 0, \qquad [\hat{k}_a, \hat{k}_b] = 0, \tag{2}$$

using the following linear transformations:

$$\hat{x}_a = \hat{q}_a - \frac{\theta_{ab}\hat{k}_b}{2}, \qquad \hat{p}_a = \hat{k}_a + \frac{\sigma_{ab}\hat{q}_b}{2}, \tag{3}$$

where summation over repeated indices (here and at other places) is understood. In the following we often take $\theta_{ab} = \theta\varepsilon_{ab}$ and $\sigma_{ab} = \sigma\varepsilon_{ab}$, where

$$\varepsilon_{ab} = \begin{cases} 1, & a < b \\ 0, & a = b \\ -1, & a > b. \end{cases} \tag{4}$$

and we assume that both θ and σ take very small values.

Suppose we have a model given by Hamiltonian $H(\hat{p}, \hat{x})$ on NC phase space in the form (1). Replacing now $(\hat{p}, \hat{x})$ in $H(\hat{p}, \hat{x})$ using the transformations (3), we get the corresponding quantum-mechanical model on the ordinary phase space $(\hat{k}, \hat{q})$, but with modified Hamiltonian $H'(\hat{k}, \hat{q})$. In this way, parameters θ and σ are transferred from the commutation relations into Hamiltonian, giving the explicit possibility of studying their influence on quantum dynamics.

3. On decoherence

Quantum decoherence is a fundamental quantum effect referring to the open systems (S) that are in unavoidable interaction with their environments (E). The composite system $S + E$ is described by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}} \tag{5}$$

where $\hat{H}_S$ and $\hat{H}_E$ represent the self-Hamiltonians of the subsystems S and E, while $\hat{H}_{\text{int}}$ represents the interaction in the system. The composite system is assumed to evolve in time according to the following equation for the density matrix:

$$\hat{\rho}_{SE}(t) = \hat{U}(t, t_\circ)\hat{\rho}_{SE}(t_\circ)\hat{U}^\dagger(t, t_\circ) \tag{6}$$

where $\hat{U}$ represents the unitary operator of (the Schrödinger) evolution in time of the isolated quantum system $S + E$.

Now, the open system state is defined by 'tracing out' the environmental degrees of freedom:

$$\hat{\rho}_S(t) = \text{tr}_E \hat{\rho}_{SE}(t). \tag{7}$$

The decoherence (or the '*environment-induced superselection rules*' [6]) effect is defined by the following two conditions [8–10]: (i) disappearance of the off-diagonal terms of $\hat{\rho}_S$ in a certain orthonormalized basis, the so-called 'pointer basis', $\{|m\rangle_S\}$:

$$\lim_{t\to\infty} {}_S\langle m|\hat{\rho}_S(t)|m'\rangle_S = 0, \qquad m \neq m' \tag{8}$$

where the limit $t \to \infty$ should not be literally understood, and (ii) robustness of the 'pointer basis' states:

$$\hat{H}_{\text{int}}|m\rangle_S|\chi\rangle_E = |m\rangle_S|\phi\rangle_E, \qquad \forall m \tag{9}$$

for arbitrary initial state of the environment, $|\chi\rangle_E$. Alternatively, decoherence is defined by the very existence of the so-called 'pointer observable' that represents the centre of algebra of the observables of the system S:

$$\hat{\Lambda}_S = \lambda_n \hat{P}_{Sn}. \tag{10}$$

The projectors $\hat{P}_{Sn}$ define an orthogonal decomposition of the Hilbert state space of the system, H_S:

$$H_S = \oplus_n H_n \tag{11}$$

such that the subspaces H_n represent the *superselection sectors*: due to the interaction with the environment, the linear superpositions of the pointer basis states belonging to the different superselection sectors are (usually quickly) destroyed–decohered. This is the reason for referring to $\hat{\Lambda}_S$ as a 'macroscopic observable', which brings the (*semi*)*classical behaviour* of the open system S [5, 6, 10]; the typical macroscopic observables are the centre-of-mass coordinates. Physically, the macroscopic observables determine the system behaviour very much like the classical variables determine behaviour of the macroscopic bodies [5, 10].

Investigating the occurrence of decoherence, one should take into account the total Hamiltonian (5). Nevertheless, the effect of decoherence *never takes place* if the interaction term $\hat{H}_{\text{int}}$ is not of a rather special kind [8, 9]. For example, condition (ii) implies diagonalizability of $\hat{H}_{\text{int}}$ in the pointer basis,

$${}_S\langle m|\hat{H}_{\text{int}}|m'\rangle_S = 0 \tag{12}$$

while $|m\rangle_S \in H_n$ and $|m'\rangle_S \in H_{n'}$, $n \neq n'$. The approximate equality in (12) gives rise to the approximate pointer basis, i.e. to the approximate equalities in (8) and (9).

Expression (12) is simultaneously a definition and the condition of existence of the pointer basis, thus defining the superselection sectors H_n. Therefore, existence of the pointer basis is a necessary, but not a sufficient condition for the occurrence of decoherence. In the following, we refer to the task of investigating the existence of the pointer basis.

Decoherence takes some time that is characterized by the so-called decoherence time, τ_D, which is typically of the form, e.g. (cf (13) below) $\tau_D \sim g_{ij}^{-1}$, where g_{ij} represent the coupling constants of the interaction in composite system. By definition, exceedingly long time intervals point out non-occurrence of decoherence, while, typically, τ_D is a very short interval (e.g. a particle of the mass 1 g, under standard macroscopic conditions and for spatial distances of the order of 1 cm, decoheres in 10^{-23} s [10]).

4. The model

We assume the linear interaction of the open system with its environment

$$\hat{H}_{\text{int}} = g_{ij}\hat{x}_i\hat{C}_j + f_{pq}\hat{p}_p\hat{D}_q, \tag{13}$$

where $\hat{x}_i$ and $\hat{p}_i$ are the position and momentum coordinates of a quantum system, for the virtually arbitrary observables of the environment. For simplicity, we omit the subscripts S and E for the system and environment, while one assumes the tensor product of the observables: e.g. $\hat{x}\hat{C} \equiv \hat{x} \otimes \hat{C}$, where $\hat{x}$ and $\hat{C}$ refer to the system and environment, respectively.

Let a charged particle with its charge e be moving in a NC plane with commutation relations (1). Using transformations (3), one obtains

$$\hat{H}_{\text{int}} = g_{ij}\left(\hat{q}_i - \frac{\theta}{2}\varepsilon_{il}\hat{k}_l\right)\hat{C}_j + f_{ij}\left(\hat{k}_i + \frac{\sigma}{2}\varepsilon_{il}\hat{q}_l\right)\hat{D}_j, \tag{14}$$

where $\hat{q}_i$ and $\hat{k}_i$ satisfy commutation relations (2).

Interaction with magnetic field $\mathcal{B}$ can be introduced by replacement $\hat{k}_l \to \hat{k}_l - e\hat{\mathcal{A}}_l$, where electromagnetic potential $\hat{\mathcal{A}}_l = \frac{\mathcal{B}}{2}\varepsilon_{lm}\hat{q}_m$. Then we have

$$\hat{H}_{\text{int}} = g_{ij}\left(\hat{q}_i - \frac{\theta}{2}\varepsilon_{il}\left(\hat{k}_l - \frac{e\mathcal{B}}{2}\varepsilon_{lm}\hat{q}_m\right)\right)\hat{C}_j + f_{ij}\left(\hat{k}_i - \frac{e\mathcal{B}}{2}\varepsilon_{im}\hat{q}_m + \frac{e\sigma}{2}\varepsilon_{im}\hat{q}_m\right)\hat{D}_j, \tag{15}$$

where magnetic field $\mathcal{B}$ is perpendicular to the NC plane. Since in the two-dimensional case $\varepsilon_{il}\varepsilon_{lm} = -\delta_{im}$, we obtain

$$\hat{H}_{\text{int}} = \hat{q}_i((1 - e\mathcal{B}\theta/4)g_{ij}\hat{C}_j - (e\mathcal{B}/2 - \sigma/2)f_{pq}\varepsilon_{pi}\hat{D}_q) + \hat{k}_i(f_{ij}\hat{D}_j - (\theta/2)\varepsilon_{pi}g_{pq}\hat{C}_q). \tag{16}$$

Existence of the magnetic field allows, in principle, the possibility of external control of the decoherence in the system modelled by (16). This is the subject of the next section.

5. The pointer basis

Here, we refer solely to investigating diagonalizability (12) of $\hat{H}_{\text{int}}$, i.e. existence of the pointer basis, that is a necessary condition for the occurrence of decoherence. In general, the occurrence of decoherence requires the analysis of the total Hamiltonian (5).

5.1. The 'macroscopic considerations'

In his classic textbook, von Neumann [11] introduced the 'coarse graining' of the position- and the momentum-axes, thus allowing the approximations of these observables. In fact, the observable $\hat{q}_i$ is approximated by the corresponding observable $\hat{\xi}_i$, while the momentum coordinate $\hat{k}_j$ is approximated by $\hat{\pi}_j$, so that one has

$$\|(\hat{q}_i - \hat{\xi}_i)|\Psi_{\mu\nu}\rangle\| \leqslant 60\Delta\hat{x}_i \tag{17}$$

and analogously for $\hat{k}_j$. The point is that, given $\Delta\hat{q}_i\Delta\hat{k}_j = \delta_{ij}\hbar/2$, the observables $\hat{\xi}_i$ and $\hat{\pi}_j$ may satisfy (a) $[\hat{\xi}_i, \hat{\pi}_j] = 0, \forall i, j$, and (b) the common eigenbasis $\{|\Psi_{\mu\nu}\}$ can be obtained from the orthonormalization of the minimal-uncertainty states, with the pure discrete spectra for both $\hat{\xi}_i$ and $\hat{\pi}_j$.

Now, model (16) can be rewritten in the form

$$\hat{H}_{\text{int}} = \hat{\xi}_i\hat{E}_i + \hat{\pi}_i\hat{F}_i + \hat{H}' \tag{18}$$

where

$$\hat{E}_i = (1 - e\mathcal{B}\theta/4)g_{ij}\hat{C}_j - (e\mathcal{B}/2 - \sigma/2)f_{pq}\varepsilon_{pi}\hat{D}_q \tag{19}$$

and

$$\hat{F}_i = f_{ij}\hat{D}_j - (\theta/2)\varepsilon_{pi}g_{pq}\hat{C}_q \tag{20}$$

while, due to (17), one may write

$$\|\hat{H}'\| \ll \|(\hat{H}_{\text{int}} - \hat{H}')\|. \tag{21}$$

Therefore, due to (21), the basis $\{|\Psi_{\mu\nu}\}\rangle$ appears as the approximate pointer basis yet at the price of large standard deviation of both $\Delta\hat{q}_i \sim 10\sqrt{\hbar}$, and $\Delta\hat{k}_j \sim 10\sqrt{\hbar}$.

Needless to say, these ‘large’ uncertainties may become useful in the macroscopic context of the theory, i.e. as regards the large, macroscopic bodies. However, if the open system (S) with characteristic dimension L is such that $L \ll \Delta\hat{q}_i$, the system position becomes completely undetermined (un-decohered)—as we learn from the experiments on the fullerene spatial interference [12].

5.2. *The coordinate coupling*

Let us assume that

$$f_{pq} = 0, \qquad \forall p, q \tag{22}$$

thus redefining model (13) to

$$\hat{H}_{\text{int}} = g_{ij}\hat{x}_i\hat{C}_j. \tag{23}$$

Without noncommutativity ($\theta = 0 = \sigma$) the position eigenbasis $|\vec{q}\rangle$, where $|\vec{q}\rangle = |\vec{x}\rangle$ when $\theta = 0$, appears as the exact pointer basis for the system. Then, obviously, $\tau_D \sim \{\min[|g_{ij}|]\}^{-1}$.

However, taking model (16) into account, condition (22) gives rise to the following redefinition of the Hamiltonian:

$$\hat{H}'_{\text{int}} = (1 - e\mathcal{B}\theta/4)g_{ij}\hat{q}_i\hat{C}_j - (\theta/2)\varepsilon_{pi}g_{pq}\hat{k}_i\hat{C}_q \approx (1 - e\mathcal{B}\theta/4)g_{ij}\hat{q}_i\hat{C}_j, \tag{24}$$

neglecting the second term since θ is assumed to take a very small value. Due to the product $e\mathcal{B}\theta$ we have now an effective coordinate coupling $(1 - e\mathcal{B}\theta/4)g_{ij}$ which can take various values by changing $\mathcal{B}$.

Then, the basis $|\vec{q}\rangle$ appears as the approximate pointer basis, for the decoherence time $\tau_D \sim |1 - e\mathcal{B}\theta/4|^{-1}\{\min[|g_{ij}|]\}^{-1}$.

5.3. The momentum coupling

Let us now assume that

$$g_{pq} = 0, \qquad \forall p, q \tag{25}$$

thus redefining model (13) as

$$\hat{H}_{\rm int} = f_{ij}\hat{p}_i\hat{D}_j. \tag{26}$$

In the absence of noncommutativity for $\mathcal{B} = 0$, the momentum eigenbasis $|\vec{k}\rangle$, where $|\vec{k}\rangle = |\vec{p}\rangle$ if $\sigma = 0$, appears to be the exact pointer basis for the system. Then, $\tau_D \sim \{\min[|f_{ij}|]\}^{-1}$.

However, taking model (16) into account, condition (25) gives rise to the following redefinition of the Hamiltonian:

$$\hat{H}_{\rm int} = -(e\mathcal{B}/2 - \sigma/2) f_{pq}\varepsilon_{pi}\hat{q}_i\hat{D}_q + f_{ij}\hat{k}_i\hat{D}_j. \tag{27}$$

This is an interesting case, indeed.

Actually, if $\mathcal{B}$ and σ are so small that the first term can be neglected with respect to the second one, model (27) reads

$$\hat{H}'_{\rm int} \approx f_{ij}\hat{k}_i\hat{D}_j \tag{28}$$

thus giving rise to $|\vec{k}\rangle$ as the approximate pointer basis, cf (26), and $\tau_D \sim \{\min[|f_{ij}|]\}^{-1}$.

However, for the very strong magnetic field $\mathcal{B}$, one obtains

$$\hat{H}''_{\rm int} \approx -(e\mathcal{B}/2) f_{pq}\varepsilon_{pi}\hat{q}_i\hat{D}_q \tag{29}$$

thus giving rise to the approximate pointer basis $|\vec{q}\rangle$, while the decoherence time $\tau_D \sim |e\mathcal{B}|^{-1}\{\min[|f_{ij}|]\}^{-1}$. The form (29) equally refers to both (16) and (26) for the sufficiently strong magnetic field.

Therefore, choosing the appropriate external magnetic field, one may obtain the mutually *exclusive* physical situations: for weak magnetic field, the open system reveals its (approximate) momentum $|\vec{k}\rangle$, while for sufficiently strong magnetic field, the system reveals its (approximate) position, $|\vec{q}\rangle$.

5.4. Comments

Even as the approximate pointer states, the states $|\vec{q}\rangle$ or $|\vec{k}\rangle$ give rise to better resolution of the system position or momentum than dealing with the 'macroscopic states' $|\Psi_{\mu\nu}\rangle$. In principle, the states $|\vec{q}\rangle$ (or $|\vec{k}\rangle$) refer to the *exact* position (or momentum) of the system, up to the Planck scale. Of course, the details in this subject depend on both the details in the interaction as well as in the self-Hamiltonian of the system $S + E$.

Interestingly enough, as long as the linear coupling is of interest, the possible occurrence of decoherence allows in principle partial distinguishing between the models discussed in the previous subsections. To this end, the situation is as follows:

(a) appearance of the pointer states $|\vec{q}\rangle$ in the presence of a strong magnetic field gives rise to either the general model (16), or model (29). Then, the decoherence time is proportional to $\mathcal{B}^{-1}$,

(b) appearance of the pointer states $|\vec{k}\rangle$ for the weak field points out the relevance of model (28),

(c) appearance of the pointer states $|\vec{q}\rangle$ independently of the magnetic field points out the relevance of model (24),

(d) non-appearance of either the pointer states $|\vec{q}\rangle$ or $|\vec{k}\rangle$ (independently of the magnetic field) points out the general model (13), i.e. (18).

5.5. Revealing noncommutativity

The conclusions of the sections 5.1 through 5.4 equally refer to both the 'commutative' models ($\theta = 0 = \sigma$) as well as to the 'noncommutative' models ($\theta \neq 0 \neq \sigma$). This is due to the very small values of θ and σ. The only instant, generally, where the two cases can be mutually distinguished, i.e. where the noncommutativity may reveal itself through the possible occurrence of the decoherence effect, is determined by the following condition:

$$\mathcal{B} = 4/e\theta \tag{30}$$

in which case model (16) reads (compare to (29))

$$\hat{H}_{\text{int}} \approx -(e\mathcal{B}/2) f_{pq} \varepsilon_{pi} \hat{q}_i \hat{D}_q = -(2/\theta) f_{pq} \varepsilon_{pi} \hat{q}_i \hat{D}_q. \tag{31}$$

In this case, the position eigenstates $|\vec{q}\rangle$ appear as the approximate pointer basis states, for the decoherence time τ_D proportional to θ. Bearing in mind a small value of θ, we conclude that this situation requires a very strong (probably physically unrealistic) magnetic field yet giving rise eventually to fast decoherence.

6. Physical analysis of the model

Of special interest is the following physical situation. An open system experiences decoherence due to the interaction of the type (13). Now, the external magnetic field is applied.

By manipulating the external magnetic field, one can in principle perform an *in situ* change of the system behaviour. For example, cf section 5.3, initially, the system reveals its momentum at the price of a totally undetermined position; in such a situation, one may observe the spatial interference of the system in analogy with the spatial interference of the fullerene molecules [12]. Now, by applying a sufficiently strong magnetic field, one may (in principle) obtain the 'appearance' of the system in front of the experimenter's eyes (at the price of the loss of information about the particle momentum). The effect may be visualized in analogy with the production of the (spatially) localized electronic states in atoms [13]. Needless to say, this effect would give rise to the loss of spatial interference, i.e. the loss of the contrast between the interference fringes—in analogy with the observation of the spatial decoherence for the fullerene molecules [14]. Interestingly enough, in the case of (30), this effect simultaneously reveals the underlying phase space noncommutativity (cf section 5.5).

The magnetic field appears as an externally controllable parameter. As distinct from the parameter T (the temperature) in the fullerene experiments [12, 14], the magnetic field allows *in situ* control of decoherence: the experimenter may, *at will*, *dynamically* change the decoherence-induced behaviour of the open system. Observing such an effect experimentally would be very interesting, indeed; work in this direction is in progress. To this end, the following requirements should be met: (i) a well-defined open system (modelled by (13)) that is confined to a plane, (ii) the possibility of making a choice of the magnetic field, provided that the strong field does not alter either the definition of the environment or the interaction in the composite system and (iii) well-defined procedures for determining the pointer states as well as the decoherence time for the open system.

7. Discussion and conclusion

The linear coupling (13) is both mathematically the simplest possible as well as the best known (investigated) in the decoherence theory so far. To this end, two remarks are in order. First, model (13) extends the usual linear position coupling, which is analysed in some detail in the literature [5, 10, 15, 16]; our model (23) reproduces the main results presented therein. The inclusion of the momentum coupling ($f_{ij} \neq 0$) in (13) is the ultimate basis of the observations and results presented in this paper. For example, we point out the appearance of the magnetic field as a parameter allowing *in situ* dynamical control of the mutually *exclusive* (complementary), the decoherence-induced behaviour of the open system. While this is *per se* interesting—the experimenter may, at will, cause 'disappearance' and 'reappearance' of the open system—this kind of the open system control essentially resembles the main idea of the 'error avoiding' methods in quantum computation theory [17–19]. Actually, driving S, e.g. into the momentum space ($|\vec{k}\rangle$ are the pointer basis states), *keeps the coherence* of the position states ($|\vec{q}\rangle$), and *vice versa*. Second, model (13) is the most general one as long as there is no linear dependence [9] in the set of the environmental observables, $\{\hat{C}_i, \hat{D}_j\}$. We assume the environment (E) is a 'macroscopic' system [5–10], for which the effects coming from the Planck scale may be ignored. This is the reason we do not take into account the phase space noncommutativity for the observables of the environment, $\hat{C}_i$ and $\hat{D}_j$.

Noncommutativity may influence the effect of decoherence *only* when the external control (here the magnetic field) is related to the noncommutativity parameters θ. In the case that (30) is fulfilled, the following physical effect takes place: instead of revealing only approximate, 'macroscopic', position *and* momentum, the particle reveals its position $|\vec{q}\rangle$ in a very short time interval proportional to θ. In all other cases—section 5.1 through 5.4—noncommutativity is hidden due to the small values of the parameters θ and σ. For example, in the case of 'momentum coupling' (section 5.3), we obtain the possibility of observing a striking physical effect: by manipulating the external magnetic field, the open system may reveal *either* its position *or* its momentum. The possible occurrence of decoherence may, in principle, partly reveal the type of interaction as long as the linear model (13) is in question. This possibility is stressed by the points (a)–(d) in section 5.4.

Therefore, in the context of model (13), by manipulating the external magnetic field, one may obtain: (a) a partial information about the type of interaction in the composite system, and/or (b) the possibility of the external, *in situ* (dynamical), control of decoherence and/or (c) the open system decoherence reveals the phase space noncommutativity. As to the point (c), the magnetic field $\mathcal{B}$ resembles the noncommutativity—cf the term $\mathcal{B}/2 - \sigma/2$ in (16). It is therefore not surprising that, for very small $\mathcal{B}$, the cases for which $\theta \neq 0 \neq \sigma$ appear essentially indistinguishable from the cases for which $\theta = 0 = \sigma$.

We conclude that, for a system that is an object of decoherence, the phase space noncommutativity may influence the occurrence of decoherence, in the sense of the possible redefinition (or even change) of the pointer basis and/or of the decoherence time. To this end, certain external control of decoherence is required, cf (30). Our analysis of a comparatively simple model (13) encourages investigation towards the more realistic physical models.

Acknowledgments

The work on this paper was supported by Ministry of Science and Environmental Protection, Serbia, under contract no 1426.

References

[1] Snyder H S 1947 *Phys. Rev.* **71** 38
[2] Douglas M R and Nekrasov N A 2001 *Rev. Mod. Phys.* **73** 977
Douglas M R and Nekrasov N A 2001 *Preprint* hep-th/0106048

[3] Szabo R J 2003 *Phys. Rep.* **378** 207
Szabo R J 2001 *Preprint* hep-th/0109162

[4] Dragovich B and Rakić Z 2004 *Theor. Math. Phys.* **140** 1299
Dragovich B and Rakić Z 2003 *Preprint* hepth/0309204
Dragovich B and Rakic Z 2005 Noncommutative quantum mechanics with path integral *Preprint* hepth/0501231

[5] Giulini D, Joos E, Kiefer C, Kupsch J, Stamatescu I-O and Zeh H D 1996 *Decoherence and the Appearance of a Classical World in Quantum Theory* (Berlin: Springer)

[6] Zurek W H 1982 *Phys. Rev.* D **26** 1862

[7] Dugić M 2000 *Quantum Comput. Comput.* **1** 102

[8] Dugić M 1996 *Phys. Scr.* **53** 9

[9] Dugić M 1997 *Phys. Scr.* **56** 560

[10] Zurek W H 1991 *Phys. Today* **44** 36

[11] von Neumann J 1955 *Mathematical Foundations of Quantum Mechanics* (Princeton, NJ: Princeton University Press)

[12] Hackermuller L *et al* 2003 *Phys. Rev. Lett.* **91** 090408

[13] Uzer T *et al* 2001 *Contemp. Phys.* **41** 1

[14] Hackermuller L *et al* 2004 *Nature* **427** 711

[15] Zurek W H, Habib S and Paz J P 1993 *Phys. Rev. Lett.* **70** 1187

[16] Caldeira A O and Leggett A J 1983 *Ann. Phys., NY* **149** 374

[17] Zanardi P and Rasetti M 1998 *Phys. Rev. Lett.* **79** 3306

[18] Lidar D A, Bacon D A and Whaley K B 1999 *Phys. Rev. Lett.* **82** 4556

[19] Beige A, Braun D and Knight P L 2000 *New J. Phys.* **2** 22.1